Measurement of the spectral absorption coefficient in the ocean with an isotropic source

Robert A. Maffione, Kenneth J. Voss, and Richard C. Honey

Closed-form equations that describe the vector irradiance from an isotropic source embedded in the ocean are rigorously derived from the steady-state radiative transfer equation. The equations are exact for a homogeneous medium and are believed to be an excellent approximation along the vertical axis for a plane-parallel ocean. The equations are solved for the absorption coefficient as a function of distance from the source. For clear ocean water, it is shown that vector irradiance measurements alone provide sufficient information for an accurate calculation of the absorption coefficient. Measurements in Pacific water of the vector irradiance from an isotropic source are presented, and the absorption coefficient is computed. The estimated value of the absorption coefficient from a linear least-squares fit to the data has a standard error of ~ 1%.

Key words: Ocean optics, absorption coefficient, isotropic source.

1. Introduction

Knowledge of the spectral absorption coefficient of oceanic waters is important to many areas of oceanography and ocean remote sensing. Yet it is still a difficult ocean-optical parameter to measure accurately and routinely. The difficulty is fundamentally due to scattering, which may cause significant errors in measurements of light attenuation from pure absorption (that is, energy conversion) processes. Nonetheless, several diverse approaches have been developed for measuring the absorption coefficient, either *in situ* or on-board ship. Some of the more notable are the reflective-tube absorption meter,[1] the integrating cavity absorption meter,[2] photoacoustic and photothermal techniques,[3,4] and *in-situ* irradiance[5,6] and radiance[7,8] meters, which rely on Gershun's law[9] to determine the absorption coefficient.

Gershun's law relates the divergence of vector irradiance to the product of the absorption coefficient and the scalar irradiance at a point in an absorbing and scattering medium. Preisendorfer[10] independently derived Gershun's law from the steady-state radiative transfer equation (RTE) and showed that the absorption coefficient could be determined by measuring the scalar irradiance and the change with depth of the net downward (vector) irradiance. He arrived at this result by considering the ocean as a medium with negligible horizontal gradients compared with its vertical structure. Experimental methods[5–8] that use this result, however, depend on the temporal invariance of the submarine daylight field and are limited to the photic zone. Furthermore, the measurements can be complicated by ship shadowing.[11,12]

Artificial light sources obviate these problems and offer several other advantages. Duntley[13] used an isotropic source to measure both the absorption and attenuation coefficients of water in Lake Winnipesaukee, New Hampshire. By illuminating the rear (relative to the detector) of a table-tennis ball with a laser beam, he was able to create a spectral underwater isotropic source. Duntley obtained the absorption coefficient by measuring the hemispherical scalar irradiance and the outward flowing irradiance from the source at long ranges. The decay of the outward irradiance provided what Duntley called a spherical diffuse attenuation coefficient that was expected to be constant at long ranges.[14] By an approximation to Gershun's law for the asymptotic region, he obtained a simple equation relating the hemispherical scalar irradiance, outward irradiance, and the spherical diffuse attenuation coefficient to the absorption coefficient.

Sorenson and Honey[15,16] designed an absorption

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meter based on a different approximation from Duntley's for the decay of irradiance from an isotropic source. Whereas Duntley used an approximation valid for the asymptotic region, Sorenson and Honey used a different approximation for the decay of irradiance within a few attenuation lengths of the source. Both approximations may be obtained from the general solution derived in this paper. Sorenson and Honey argued that within a few mean free paths, the outward irradiance would decay geometrically as $1/r^2$ and exponentially as $\exp(-ar)$, where a is the absorption coefficient, and r is the distance from the source. Their instrument measured the outward irradiance at two separate distances from an isotropic source. Because the ratio of the two measurements cancels the radiometric units, their instrument did not require any absolute radiometric calibration. The construction and the testing of this device are described in detail by Gilbert *et al.*[17]

The technique of Sorenson and Honey was recently used to measure the absorption coefficient in an experiment to test the closure property in ocean optics.[18] In this experiment, two spectral irradiance detectors looked down at an isotropic source and two looked up, away from the source. The distance between the source and detectors was varied, and measurements were taken over a range of separations from approximately 2 to 40 m. Whereas the Sorenson and Honey instrument measured the irradiance at only two distances from the source, the closure experiment provided a series of measurements of vector irradiance as a function of range. A least-squares fit of the measurements to the appropriate equation, derived in this paper, yields the absorption coefficient with an accurately quantified error. Measurements of the variation of the absorption coefficient with depth were obtained by changing the depth of the light source and detectors.

2. Theoretical Development

A. Derivation of the Vector and Scalar Irradiance from an Isotropic Source Embedded in the Ocean

Nearly all techniques developed for solving the RTE treat the ocean as a horizontally homogeneous, vertically inhomogeneous optical medium. If the only source for the submarine light field is the penetration of solar radiation, then the spatial distribution of radiance in the ocean will be a function of depth only. The reduction to one spatial coordinate greatly simplifies manipulation of the RTE. For an isotropic source embedded in the ocean, however, the radiance will clearly depend on all three spatial coordinates. It is for this reason that the most difficult standard problems encountered in radiative transfer theory are point-source problems. Aside from the approximate solutions already mentioned, some general formulations have been attempted, such as Preisendorfer's discrete-space method[19] and results from diffusion theory.[20]

A general, closed-form solution for the vector and scalar irradiance distribution from an isotropic source in the ocean is now derived from the RTE. It is valid at any depth z on axis with the source for a plane-parallel ocean and is exact everywhere for a homogeneous medium. This solution lends itself quite readily to approximations for off-axis irradiance distributions in a plane-parallel medium and to the irradiance distribution from a Lambertian (cosine) source. It is therefore applicable not only to measurements of the absorption coefficient, but also to bio-optical modeling of the irradiance distribution from bioluminescent organisms.[21]

Let $L = L(\mathbf{r}, \hat{\xi})$ denote the radiance at a point $\mathbf{r} = (x, y, z)$ in the unit vector direction $\hat{\xi}$ in a Cartesian coordinate system. Assuming no internal sources and no cross-wavelength effects, i.e., fluorescence and Raman emission, the steady-state RTE may be expressed as

$$(\hat{\xi} \cdot \nabla)L(\mathbf{r}, \hat{\xi}) = -c(\mathbf{r})L(\mathbf{r}, \hat{\xi}) + L^*(\mathbf{r}, \hat{\xi}), \tag{1}$$

where the operator

$$(\hat{\xi} \cdot \nabla) = (\cos\theta_x, \cos\theta_y, \cos\theta_z) \cdot \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \tag{2}$$

takes the derivative of $L(\mathbf{r}, \hat{\xi})$ in the direction $\hat{\xi}$. The first term on the right represents the loss of radiance along an infinitesimal path length in the direction $\hat{\xi}$, and $c(\mathbf{r})$ is the beam attenuation coefficient at point $\mathbf{r}$. The last term on the right is the so-called path function, which gives the increase in radiance per unit path length that is due to scattering into the direction $\hat{\xi}$ from all other directions $\{\hat{\xi}'\}$. If $\beta(\mathbf{r}, \hat{\xi} \cdot \hat{\xi}')$ deontes the volume scattering function and $d\Omega(\hat{\xi}')$ denotes the infinitesimal solid angle in the direction $\hat{\xi}'$, then the path function is given explicitly by

$$L^*(\mathbf{r}, \hat{\xi}) = \int_{4\pi} L(\mathbf{r}, \hat{\xi}')\beta(\mathbf{r}, \hat{\xi} \cdot \hat{\xi}')d\Omega(\hat{\xi}'). \tag{3}$$

The quantities L, c, and β are assumed to be spectral, although their explicit dependence on wavelength is not shown.

Integrating Eq. (1) over 4π sr gives the general form of Gershun's equation:

$$-\nabla \cdot \mathbf{E}(\mathbf{r}) = a(\mathbf{r})E_0(\mathbf{r}). \tag{4}$$

Here a is the spectral absorption coefficient and E_0 is the scalar irradiance, which is given by

$$E_0(\mathbf{r}) = \int_{4\pi} L(\mathbf{r}, \hat{\xi})d\Omega(\hat{\xi}). \tag{5}$$

Note that, in general, vector irradiance $\mathbf{E}$ has three components: E_x, E_y, E_z, where

$$\begin{aligned} E_x &= 2\pi \int_0^{\pi} L(\mathbf{r}, \hat{\xi})\cos\theta_x \sin\theta_x d\theta_x \\ &= 2\pi\left[\int_0^{\pi/2} L(\mathbf{r}. \hat{\xi})\cos\theta_x \sin\theta_x d\theta_x \right. \\ &\quad \left. - \int_{\pi/2}^{\pi} L(\mathbf{r}, \hat{\xi})|\cos\theta_x|\sin\theta_x d\theta_x\right] \\ &= E_{+x} - E_{-x}, \qquad \text{etc.} \end{aligned} \tag{6}$$

Thus E_x represents the difference between the irradiance that flows in the positive x direction and the irradiance that flows in the negative x direction.

Under the assumption of a horizontally homogeneous ocean illuminated by solar radiation, the partial derivatives $\partial E_x/\partial x$ and $\partial E_y/\partial y$ in Eq. (4) will be 0, and Gershun's equation simplifies to

$$-\frac{\partial E_z(z)}{\partial z} = aE_0(z). \tag{7}$$

which is the result used most often in ocean optics. In principle then, the absorption coefficient can be determined by measuring the change in the vector irradiance and the scalar irradiance with depth. In practice it is quite difficult, and little data have been published on the measured spectral absorption coefficient from Gershun's law [Eq. (7)]. Significant errors can be introduced in computing the derivative of E_z, especially in regions where the water is not vertically homogeneous. Measurements are restricted to the daytime and to depths where the ambient light is strong enough for the sensors to detect. Ship shadowing is another problem that has already been mentioned. But a major reason why researchers do not routinely apply Gershun's law is that the scalar irradiance is rarely measured. Sometimes, however, the scattering properties of the water are estimated, which then allows one to estimate the absorption coefficient from Gershun's law.

For an isotropic source embedded in the ocean, the divergence of vector irradiance $\mathbf{E}$ will be a function of all three spatial coordinates. Assuming that the isotropic source is the only source for the light field, a natural coordinate system for its description are spherical coordinates. The three components of the vector irradiance are then E_r, E_θ, and E_ϕ, which correspond to the respective unit orthogonal directions, $\hat{r}$, $\hat{\theta}$, and $\hat{\phi}$, shown in Fig. 1. The source is taken to be at the origin, and the z axis is the vertical direction in the ocean. In this coordinate system, Gershun's law (Eq. 4) appears as

$$-aE_0 = \frac{1}{r^2}\frac{\partial(r^2E_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, E_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial E\phi}{\partial\phi}. \tag{8}$$

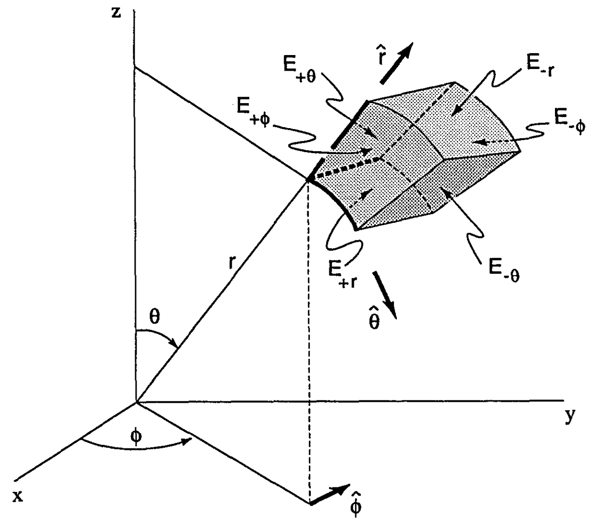

Fig. 1. Spherical components of vector irradiance. The three components of the vector irradiance are the differences of the irradiances incident upon the opposing faces.

Clearly, for a plane-parallel medium in which the planes are oriented perpendicular to the z axis, E_ϕ and $\partial E_\phi/\partial\phi$ must be 0 for all r, θ, and ϕ. Along the z axis ($\theta = 0$), it should also be clear that $E_\theta = 0$. Equation (8) then simplifies to

$$\begin{aligned} -aE_0 &= \frac{1}{r^2}\frac{\partial(r^2E_r)}{\partial r} + \frac{1}{r}\frac{\partial E_\theta}{\partial\theta} \\ &= \frac{\partial E_r}{\partial r} + \frac{1}{r}\left(2E_r + \frac{\partial E_\theta}{\partial\theta}\right). \end{aligned} \tag{9}$$

In all practical cases of interest, $\partial E_\theta/\partial\theta$ will be negligible compared with E_r, and thus

$$-aE_0 = \frac{\partial E_r}{\partial r} + \frac{2E_r}{r}, \tag{10}$$

which is everywhere exact for a homogeneous medium and an excellent approximation along the vertical axis for a plane-parallel ocean.

With the introduction of the radial average cosine,

$$\bar{\mu}_r(r) = \frac{E_r(r)}{E_0(r)}, \tag{11}$$

which is similar to the commonly used vertical average cosine, Eq. (10) may be expressed in the form

$$\frac{dE_r(r)}{E_r(r)} = -\frac{a(r)}{\bar{\mu}_r(r)}dr - \frac{2}{r}dr. \tag{12}$$

Integrating from r_0 to r and solving for $E_r(r)$ gives

$$E_r(r) = E_r(r_0)\frac{\exp\left[-\int_{r_0}^{r}\frac{a(r)}{\bar{\mu}_r}dr\right]}{(r/r_0)^2}. \tag{13}$$

If $R \leq r_0 \ll 1/c$, where R is the radius of the source, then the initial condition, $E_r(r_0)$, is

$$E_r(r_0) = \frac{\Phi_{r_0}}{4\pi {r_0}^2}, \tag{14}$$

where Φ_{r_0} is the total radiant power at r_0. Thus, for $r = z$, i.e., $\theta = 0$, the solution for the vector irradiance from an isotropic source embedded in a plane-parallel ocean is

$$E_r(z) = \frac{\Phi_{z_0}}{4\pi z^2} \exp\left[-\int_{z_0}^{z} \frac{a(z)}{\bar{\mu}_r(z)} \mathrm{d}z\right]. \tag{15}$$

Consider now a homogeneous, *nonscattering* medium. Then a is constant, and Eq. (15) simplifies to

$$E_r(z) = \frac{\Phi_{z_0}}{4\pi z^2} \exp(-a\Delta z), \tag{16}$$

where $\Delta z = z - z_0$. The inclusion of scattering gives

$$E_r(z) = \frac{\Phi_{z_0}}{4\pi z^2} \exp\left[-a \int_{z_0}^{z} \frac{\mathrm{d}z}{\bar{\mu}_r(z)}\right]$$
$$= \frac{\Phi_{z_0}}{4\pi z^2} \exp(-a\bar{z}), \tag{17}$$

where $\bar{z}$ is Schellenberger's mean light path,[22] *viz.*,

$$\bar{z} = \int_{z_0}^{z} \frac{\mathrm{d}z}{\bar{\mu}(z)}, \tag{18}$$

which appears in Stavn's three-parameter model[23] and is a similarly useful concept in the present context. A comparison of Eqs. (17) and (18) shows that the net effect of scattering on the irradiance is completely described by the mean light path $\bar{z}$. For any scattering medium, $\bar{\mu}_r < 1$ and thus $\bar{z} > \Delta z$, which means that there will be a decrease in the net irradiance at z by a factor $\exp[-a(\bar{z} - \Delta z)]$. Another way of expressing this is to rewrite Eq. (17) as

$$E_r(z) = \frac{\Phi_{z_0}}{4\pi z^2} \exp[-a(\Delta z + \delta z)], \tag{19}$$

where $\delta z = \bar{z} - \Delta z$ is defined as the mean increase in the light path that is due to scattering (see Fig. 2).

B. Solution for the Absorption Coefficient

Equation (15) is easily solved for $a(z)$, giving

$$a(z) = \bar{\mu}_r(z)\left[-\frac{1}{E_r(z)}\frac{\mathrm{d}E_r(z)}{\mathrm{d}z} - \frac{2}{z}\right]$$
$$= \bar{\mu}_r(z)\left[K_{E_r}(z) - \frac{2}{z}\right], \tag{20}$$

where K_{E_r} is defined as the diffuse attenuation coefficient for vector irradiance from an isotropic source. This follows the usual convention for defining irradiance attenuation coefficients, *viz.,*

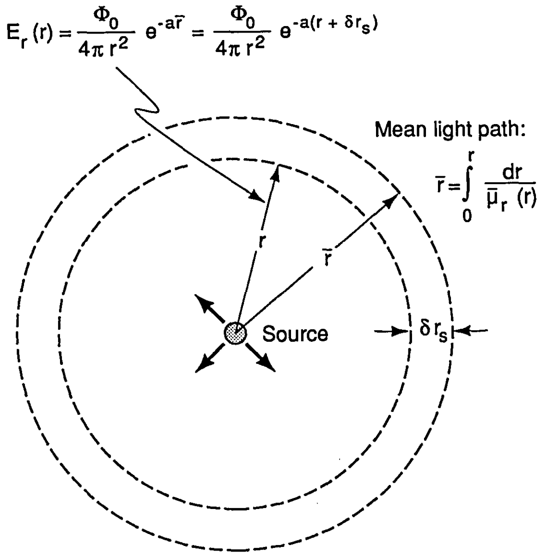

Fig. 2. Vector irradiance from an isotropic source embedded in a homogeneous scattering and absorbing medium.

$$K_{E_r}(z) \equiv \frac{-1}{E_r(z)}\frac{\mathrm{d}E_r(z)}{\mathrm{d}z}.$$

From the definition of the irradiance attenuation coefficient, the vector irradiance may be expressed as

$$E_r(z) = E_r(z_0)\exp\left[-\int_{z_0}^{z} K_{E_r}(z)\mathrm{d}z\right]. \tag{21}$$

Solving Eq. (20) for K_{E_r} gives

$$K_{E_r} = \frac{a(z)}{\bar{\mu}_r(z)} + \frac{2}{z}. \tag{22}$$

Substituting this expression into Eq. (21) gives the same result as Eq. (13) for the vector irradiance, as it should. Note that for a homogeneous ocean, as $z \to \infty$, $K_{E_r} \to K_E = a/\bar{\mu}$, where K_E is the well-known diffuse attenuation coefficient for vector irradiance defined for solar, i.e., plane wave, illumination.

Since the light field from an isotropic source approaches a plane wave in the far field, all the results derived here must approach, at large distances from the source, the well-known relations in ocean optics that are derived for plane waves incident upon the water's surface. Consider again Eq. (20). As z increases, the second term on the right eventually becomes negligible compared with K_{E_r}, resulting in the approximation

$$a(z) \cong \bar{\mu}_r(z)K_{E_r}(z), \tag{23}$$

which is identical to Preisendorfer's solution[10] for a plane-parallel ocean illuminated by solar radiation. Again for a homogeneous ocean, $K_{E_r} \to K_E \to K_\infty$, where K_∞ is the asymptotic diffuse attenuation coefficient. Because the equivalencies are achieved at large distances from the source, theoretical proofs[14,24] of the existence of a submarine asymptotic daylight field apply just as well to an isotropic source embedded in the ocean. From Eq. (20) it is seen that the condition for the asymptotic field from an isotropic source in the ocean is $2/K_{E_r} \ll z$. This condition will always be met for large enough values of z, since K_{E_r} is bounded and, for homogeneous water at least, eventually approaches a constant. Because the light field from an isotropic source is truly axially symmetric, Zaneveld and Pak's derivation,[25] which relates the asymptotic radiance distribution and its derivative to the beam attenuation coefficient and the volume scattering function, also applies. This opens up the possibility for testing these theories. Furthermore, measurements in homogeneous water can more easily be made since the source–receiver system can be lowered to depths where the water column between the source and receiver is found to be homogeneous.

Experimentally, measurements are taken at discrete points z_n, so that $K_{E_r}(z_n)$ would be calculated from

$$K_{E_r}(z_n) = \frac{\ln\left[\dfrac{E_r(z_{n-1})}{E_r(z_{n+1})}\right]}{z_{n+1} - z_{n-1}}, \tag{24}$$

where $z_{n+1} > z_n > z_{n-1}$. The operational form of Eq. (20) is then

$$a(z_n) = \bar{\mu}_r(z_n)\left\{\frac{\ln\left[\dfrac{E_r(z_{n-1})}{E_r(z_{n+1})}\right]}{z_{n+1} - z_{n-1}} - \frac{2}{z_n}\right\}. \tag{25}$$

To determine the absorption coefficient at a point z_n, three physical quantities must be measured: the vector irradiance E_r, the average cosine $\bar{\mu}_r$, and the distance z between the source and the detectors. Measuring vector irradiance and distance is straightforward and can be performed accurately.[18] One way to determine $\bar{\mu}_r$ is to measure the scalar irradiance E_0 and compute the ratio E_r/E_0. Scalar irradiance meters, however, are much more difficult to construct and use[5,6] and are subject to various sources of error. The average cosine could be estimated from measurements of the point-spread function[26] (PSF) from the isotropic source. Since the PSF is the radiance distribution, $L(z, \theta)$, normalized to the source power, $\bar{\mu}_r$ is estimated from

$$\bar{\mu}_r(z) \cong \frac{\displaystyle\int_{\epsilon\to 0}^{\theta_{max}} L(z, \theta)\cos\theta \sin\theta \, d\theta}{\displaystyle\int_{\epsilon\to 0}^{\theta_{max}} L(z, \theta)\sin\theta \, d\theta}. \tag{26}$$

The ratio cancels the radiometric units, so that no absolute calibration of the camera and source is necessary. Measurements by Voss and Chapin[26] and Maffione *et al.*[18] of the PSF in ocean waters show that the PSF falls off by more than 3 orders of magnitude within the first 10 deg, which is expected since scattering in the ocean is highly peaked in the forward direction.[27] Measurements by Wilson[28] of the radiance distribution from a point source in an aluminum hydroxide suspension show that the PSF continues to drop off by another 2 orders of magnitude from 10 to ~50 deg. Thus an accurate estimate of $\bar{\mu}_r$ can be obtained by measuring the PSF over forward angles, i.e., $\theta_{max} \leq 90°$, except perhaps in murky water or at asymptotic distances where a larger fraction of the scattered light is contained at larger angles. Underwater camera systems have become a well-developed technology,[7,18,26,29] which in many circumstances makes their use preferable to the use of scalar irradiance meters because the measurement of the radiance distribution provides additional information about the optical properties of the water.

To the authors' knowledge, no data have been published on $\bar{\mu}_r(z)$ as a function of distance z from the source. Nonetheless, its limiting values can be inferred. From its definition, it is clear that $\bar{\mu}_r(z) \to 1$ as $z \to z_0$. In clear homogeneous water, $\bar{\mu}_r(z)$ should gradually and monotonically decrease and must eventually approach its asymptotic value as $z \to \infty$. In the asymptotic limit, K_{E_r} must approach K_∞, the asymptotic diffuse attenuation coefficient. With the Wilson–Honey relationship[28] for the asymptotic diffuse attenuation coefficient, $K_\infty = c - \frac{5}{6} b$, where b and c are the total scattering and the beam attenuation coefficients, respectively, the following equation is easily derived from relation (23):

$$\bar{\mu}_\infty = \frac{1 - \omega_0}{1 - \frac{5}{6}\omega_0}, \tag{27}$$

where $\omega_0 = b/c$ is the single scattering albedo. Measurements of the optical properties of ocean waters at 530 nm by Petzold[30] show that ω_0 varies from ~0.3 in the clearest waters to ~0.9 in murky (e.g., harbor) waters. $\bar{\mu}_\infty$ is thus bounded approximately by $0.4 < \bar{\mu}_\infty < 0.9$, with the lower value for murky waters and the upper value for the clearest waters.

Since, for clear ocean water, $\bar{\mu}_\infty$ is ~0.9, then it should be expected that $\bar{\mu}(z)$, which starts out at 1 at z_0, will remain close to 1 for at least several attenuation lengths. Clear ocean water measurements of the vector irradiance from an isotropic source will therefore decay, according to Eq. (16), to a good approximation over distances extending to several attenuation lengths, assuming the water column is relatively homogeneous, i.e., a is constant.

Let $\tilde{E} = k_e E$ denote the signal from the irradiance detector in, say, digital counts or volts, where k_e determines the response to the irradiance E at the detector. For all practical purposes, z_0 may be taken to be at the origin, and Eq. (16) may then be expressed in the form

$$\tilde{E} = \frac{k_e \Phi_0 \exp(-az)}{4\pi z^2}, \quad (28)$$

where Φ_0 is now the total power emitted by the source. Multiplying through by z^2 and taking the natural logarithm gives

$$\ln[z^2 \tilde{E}(z)] = \ln\left(\frac{k_e \Phi_0}{4\pi}\right) - az. \quad (29)$$

A plot of the left-hand side versus z yields a line whose slope is the negative of the absorption coefficient. Because the offset, $\ln(k_e\Phi_0/4\pi)$, is not needed to determine the slope, no absolute calibration is necessary.

At large distances from the source or in water where scattering is high, z in Eq. (29) should be replaced by $\bar{z}$, Schellenberger's mean light path [Eq. (18)]. But, this requires knowing $\bar{\mu}_r(z)$. In many situations, however, the error in using z instead of $\bar{z}$ in estimating a with Eq. (29) will be small since $\bar{z} = z + \delta z = z(1 + \delta z/z)$. That is, even though δz increases as z increases, the ratio $\delta z/z$ remains small.

3. Data

Measurements of the vector irradiance $E_r(z)$ from an isotropic source were made in waters off the coast of southern California during the Ocean Optics cruise in August 1990, sponsored by the Office of Naval Research. The experimental technique and instrumentation are described in Maffione *et al.*[18] and Brown *et al.*[31] In addition to the vector irradiance measurements, simultaneous measurements of the apparent radiance of the source and the PSF were made. The decay of the apparent radiance of the source provided the beam attenuation coefficient, and the PSF gave information on the forward-scattering properties of the water.

Measurements were made at a wavelength of 465 ± 15 nm. The data presented here contain the largest source-to-detector separation profile, which is from the last run on 26 August. In this run, the detectors and cameras were held at a depth of approximately 72 m, and the distance to the isotropic source was varied by lowering the source from ~78 to 140 m. Measurements were taken at approximately 5-m increments.

Figure 3 shows a log-linear profile of the vector irradiance. The bottom abscissa is the distance between the source and detectors. The top abscissa shows the depth of the source. The units for the irradiance are in digital counts. The absorption coefficient a and the offset term $\ln(k_e\Phi_0/4\pi)$ were calculated from a linear least-squares fit to the data with Eq. (29). The curve in the plot was computed from Eq. (16) with the values of the linear fit for a and $k_e\Phi_0/4\pi$. The absorption coefficient, which is given by the absolute value of the slope, was computed to be 0.0337 m^{-1}. The standard error of the slope was 0.0003 m^{-1}. This gives a percentage error in a of ~1%.

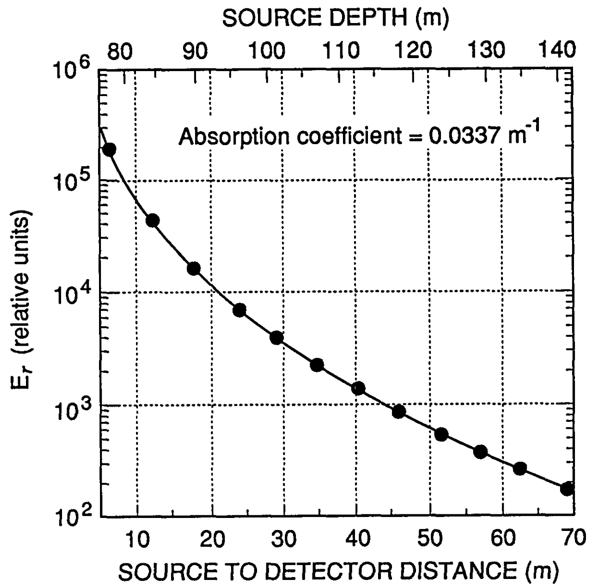

Fig. 3. Vector irradiance measurements from an isotropic source taken on 26 August 1990 off the coast of southern California.

From measurements of the apparent radiance of the source, which were made with a CCD camera,[18] the beam attenuation coefficient was found to be $c = 0.077\ m^{-1}$. Therefore $b = c - a = 0.043\ m^{-1}$, and thus the single scattering albedo, $\omega_0 = b/c$, was 0.56, which gives $\bar{\mu}_\infty \cong 0.8$ from Eq. (27). The change in $\bar{\mu}_r(z)$ from its value of 1 at the source was therefore probably small, which is one reason why there was such a good fit to the data. Nonetheless, it would be quite interesting, in future experiments, to determine $\bar{\mu}_r(z)$ and its effect on the computation of a.

4. Summary and Conclusions

Closed-form solutions for the vector irradiance from an isotropic source embedded in the ocean were derived from the RTE. The solutions are exact everywhere for a homogeneous optical medium and are believed to be an excellent approximation along the vertical axis in a plane-parallel ocean, although this has yet to be rigorously demonstrated. The vector irradiance equations were solved for the absorption coefficient. Operational equations for the absorption coefficient were also given. The appropriate solution depends on the experimental setup and assumptions about the clarity and homogeneity of the water column.

Measurements of the vector irradiance from an isotropic source were performed off the coast of southern California, and some of the data were presented. The instrumentation used to carry out the measurements consisted of upward- and downward-facing irradiance detectors, a CCD camera system, and an isotropic source. Neither an absolute radiometric calibration nor an intercalibration between the detectors and camera were needed. For clear, homogeneous water, it was shown that irradiance measurements only are required to determine the absorption coefficient accurately. The absorption coefficient was computed from a profile of the vector irradiance. The percent error of the estimate was less than 1%.

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